Lieber

Stress concentrations around reinforced circular cutouts in matalite panels in tension.

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STRESS CONCENTRATIONS
AROUND
REINFORCED CIRCULAR CUTOUTS
IN
METALITE PANELS IN TENSION

A Thesis
Submitted to the Graduate Faculty
of the
University of Minnesota

by
JAMES C. LIEBER

In Partial Fulfillment of the Requirements
for the
Degree of Master of Science in Aeronautical Engineering
May, 1952

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SUMMARY

This investigation is concerned with the problem of obtaining a means for designing reinforcing doublers for circular cutouts in tension panels. Tension panels include sandwich type material on which the doublers may be riveted or bonded.

The analysis was limited to a plane sheet of infinite width. All tests were performed on Metalite with .032 in. faces and .5 in. balsa cores.

From previously determined formulas, a chart was developed which expresses the maximum stress concentration at a cutout as a function of the ratios of doubler radius to cutout radius and of sheet thickness to doubler thickness. The results of the tests on non-infinite Metalite verified this chart and also demonstrated that bonding is superior to riveting as a means of attachment of doublers to Metalite.

INTRODUCTION

The modern aircraft has become faster, heavier, and more dependent on a large supply of fuel. The structures are complex and have more cutouts for accessory holes, doors, windows, etc. It is necessary, therefore, that these cutouts be reinforced in such a manner as to provide maximum stress reduction at a minimum weight. At the present time this is more or less a "cut and try" process.

Sandwich type material is one of the new developments for aircraft. It consists of two thin sheets of metal with a relatively thick, non-homogeneous, core of low density material bonded together. Reinforcing doublers may be internally bonded or externally riveted.

This investigation was undertaken with two thoughts in mind: First, to derive a means for determination of the proper size of doubler to be used for a given cutout, and; Second, to compare the bonded doublers with the riveted doublers as to their effectiveness in the reduction of stress concentration.

The analysis was limited to the condition of uniform tension applied to a plane sheet of infinite width having a centrally located circular cutout with concentric doubler.

The tests were made on Metalite panels fabricated by Chance Vought Division of United Aircraft Corp. The panels had .0[illegible] in. faces of 75S-T Alclad and a .5 in. core of 7-9 lb/ft.^{3} density end grain balsa. They were bonded with Redux. Internal doublers of .0[illegible] in. and .064 in. and an external riveted doubler of .064 in. were used. Cutouts varied from 2 in. to 5 in. while the doublers were 9 in. in diameter. A uniform condition of tension was attempted but not achieved for the tests.

The author wishes to express his appreciation to Professor Joseph A. Wise for his valuable advice and assistance on all phases of the investigation, and to Mr. [illegible] McDonough and Mr. W. C. Broding of Chance Vought Aircraft Division for their assistance in helping formulate the problem and for furnishing the Metalite test panels.

This project was carried out during the academic year 1951-1952 at the University of Minnesota under the supervision of Professor Joseph A. Wise, thesis adviser.

THEORETICAL ANALYSIS

If a plane sheet of infinite width has a centrally located hole and is subjected to a uniform tensile load, it will be found that a maximum stress concentration occurs at the border of the hole at an angle of 90° from the direction of the applied tension. From St. Venant's principle, it can be concluded that the stress increase diminishes rapidly away from the hole.

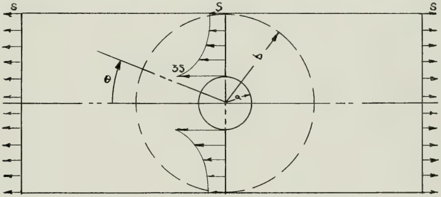

Figure 1

Considering Fig. (1); as given in Ref. (a), if b is large in comparison to a (five times or more), it can be assumed that stresses at radius b are essentially the same as in a plate with no hole and are given as;

$$(\sigma_r)_{r=b} = \frac{1}{2} S + \frac{1}{2} S \cos 2\theta$$

$$(\tau_{r\theta})_{r=b} = -\frac{1}{2} S \sin 2\theta$$

These forces acting on a ring of radii a and b have been shown to give the stress distribution within the ring expressed by

the equations:

$$\sigma_r = \tfrac{1}{2} S \left(1 - \frac{a^2}{r^2}\right) + \tfrac{1}{2} S \left(1 + \frac{3a^4}{r^4} - \frac{4a^2}{r^2}\right) \cos 2\theta$$

$$\sigma_\theta = \tfrac{1}{2} S \left(1 + \frac{a^2}{r^2}\right) - \tfrac{1}{2} S \left(1 + \frac{3a^4}{r^4}\right) \cos 2\theta$$

$$\tau_{r\theta} = -\tfrac{1}{2} S \left(1 - \frac{3a^4}{r^4} + \frac{2a^2}{r^2}\right) \sin 2\theta$$

It can be found that when $r = a$ and $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ that $\sigma_r = \tau_{r\theta} = 0$ and that $\sigma_\theta = \sigma_{\theta \max} = 3S$.

It is this stress concentration of 3S that must be reduced by a reinforcing doubler. As shown in Ref.(b), the above formulas were further developed for a plate with a doubler. These formulas with their constants are contained in Appendix A.

Examination of the formulas in App. A shows them to be quite lengthy and difficult to use. Further examination reveals that dimensionless coefficients may be employed in the formulas and their complexity may be reduced.

Because the point at the hole border at $\theta = \frac{\pi}{2}$ is the principle point in question, the formula for $\sigma_{\theta i}$, as given in App. A, was the one selected for further development.

The following symbols were used;

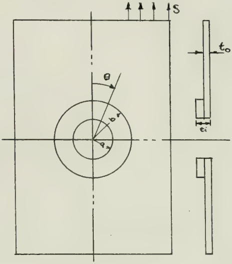

Figure 2

a = radius of cutout

b = radius of doubler

to = thickness of sheet

ti = thickness of sheet and doubler

$k = \frac{b}{a}$

$q = \frac{to}{ti}$

S = applied stress

σ_{θ_a} = stress at hole due to $\frac{1}{2}S$ component of radial stress

σ_{θ_b} = stress at hole due to $\frac{1}{2}S$ Cos 2θ component of radial stress and $-\frac{1}{2}S$ Sin 2θ component of shear stress

σ_θ = total stress at hole in terms of S

μ = Poisson's ratio

$E = (3 - \mu)$ $\quad H = (1 + \mu)^2$ $\quad L = (\mu - \mu^2)$

$F = (1 + \mu)$ $\quad I = (3 + 2\mu - \mu^2)$ $\quad M = (3 + \mu^2)$

$G = (1 - \mu^2)$ $\quad J = (5 - 2\mu + \mu^2)$

If the above dimensionless ratios, q and k, are substituted in the $\sigma_{\theta i}$ formulas for $\theta = \frac{\pi}{2}$ and r = a, the following formulas are the result.

$$\sigma_{\theta_a} = \frac{-2S}{F\left[\frac{1}{k^2}\left(\frac{1}{q}-1\right) - \left(\frac{1}{q}+\frac{G}{H}\right)\right]}$$

$$\sigma_{\theta_b} = \frac{-8\left[\left(\frac{3F}{k^9}-\frac{2F}{k^6}\right)\left(\frac{1}{q}-1\right)-\left(\frac{F}{q}+E\right)\right]S}{\frac{4}{k^2}\left(\frac{2L}{q}-\frac{I}{q^2}+M\right)-\left(\frac{6}{k^9}-\frac{4}{k^6}\right)\left(\frac{2G}{q}-\frac{I}{q^2}+H\right)+\frac{1}{k^8}\left(\frac{I}{q^2}-\frac{2I}{q}+I\right)+\left(\frac{2J}{q}+\frac{I}{q^2}+I\right)}$$

$$\sigma_\theta = \sigma_{\theta_a} + \sigma_{\theta_b}$$

A plot was constructed for σ_θ vs. k for various values of q. It is illustrated as Fig. (7). This chart provides a convenient means for obtaining maximum stress at a hole for given dimensions of the sheet and doubler.

The formulas of App. A. were considered further for stresses in the sheet outside of the doubler. It was found that, even for very narrow doublers, if the stress at the hole is not reduced below S, the stress at the minimum section will never be greater than the stress at the hole. The stress at the hole is then the maximum and need be the only one to consider. Fig. (7), therefore, is the only plot that need be made from the stress formulas.

One of the prime considerations in the design of a doubler is the question of weight saving. Fig. (3) can be used to calculate the weights of doublers that provide the same stress reduction. The below formula can be used for calculating the weight of a doubler.

$$\text{weight} = (\text{density})\ (\text{volume})$$

$$w = \rho\,(\pi)(b^2 - a^2)(ti - to)$$

$$w = \rho\pi\, a^2 to(k^2 - 1)(\frac{1}{q} - 1)$$

Using Fig.(3) typical cases can be evaluated. It will be found that the thicker doubler with the smaller radius will be the lightest. In the region of small k, however, the danger of inadequate stress reduction would be prevalent.

EQUIPMENT AND PROCEDURE

The test specimens consisted of four Metalite panels fabricated by Chance Vought Aircraft Division of United Aircraft Corporation. The faces of the panels were .032 in. 75ST-6 Alclad. The core was made of 7 - 9 lbs/ft.^{3} end grain balsa. All panels were bonded with Redux. As received each panel measured 18 in. x 27 in. with a 1½ in. x 18 in. 75ST aluminum insert at each end. One panel contained no reinforcements; two panels had 9 in. diameter, centrally positioned, internally bonded, doublers of .032 in. and .064 in. 75ST-6 Alclad. The remaining panel had a 9 in. diameter, .064 in. aluminum external riveted doubler on each face. The rivets used were Huck Brazier Head Blind Rivet, [illegible], 1/8 in. diameter.

To prepare the panels for testing, a central hole of 3 in. diameter was cut in each. For attaching to the supporting apparatus, 20 bolt holes of 3/16 in. were drilled in each end. Baldwin SR-4 electric strain gages were cemented to the panels at various locations. The types of gages were, A-7, A-11, A-8, and A-12. The very small gage lengths of most of these gages made them very adaptable for placement near the cut-out boundary.

Figs. (4),(5),(6),(7), and (8) are sketches and photographs of the test specimens giving pertinent dimensions and strain gage locations.

The principle testing apparatus was a Southwark Emery testing machine with Tate-Emery load indicator manufactured by the Baldwin Southwark Division of Baldwin Locomotive Works, Philadelphia, Pa. The strain indicating device was the Anderson 24 point Strainmeter, Model 101, manufactured by Arthur A. Anderson, Springdale, Conn. A standard galvanometer and two 1½ volt dry cells completed the measuring apparatus. SR-4 strain gages, of the types used on the panel, were mounted on a small piece of Metalite and were used for temperature compensating gages.

In an attempt to obtain a uniform distribution of the load across the top of the panels, an attachment rig was fabricated from steel. The load from the test machine was transmitted to two 3 in. channel beams which in turn transmitted the load to two 6 in. x ½ in. plates through four 2 in. x 3/8 in. steel bars. The plates were bolted to the panels through the aluminum inserts with 28 3/16 in. bolts. Fig.(9) shows the complete test setup and Fig.(10) gives a larger view of a panel and its attachment gear.

A panel was mounted in the testing machine and the strain gage leads from the test and compensating gages were connected to the strainmeter. The load was run up to 20,000 lbs. and down to zero to set all gages. A load of 100 lbs. was then put on the panel and all strain readings were adjusted to zero.

The machine was run up to 6,100 lbs. and readings from the top six gages were taken. It was desired to make these gages read equally in order to produce a state of uniform tension across the plate. Since they were never equal, the load was run down again and shims were placed under the bolt supports of the various 2 in. loading bars. The strainmeter would be re-zeroed at 100 lbs. and a load of 6,100 lbs. would be impressed again. This process would be repeated as many as 20 times until the best possible loading was obtained. The uniform tension condition, however, was never attained.

With the final shimming completed, all gages were re-zeroed for 100 lbs. and the first load of 6,100 lbs. was set on the machine. At this loading all gages were read. The loadings were then increased to 12,100, 18,100, 24,100, 30,100, and 36,000 lbs. and a reading of all gages made for each load. The loading was then reduced to 100 lbs. and the zero reading checked. It was found that for the riveted panel the zero readings were off a considerable amount. This was due to the fact that the doubler was not fabricated from 75ST-6 Alclad as were all of the other doublers. This run was repeated using a maximum load of 25,100 lbs. For this loading the zero readings were satisfactory. The panel with no doubler was limited to a 20,100 lb. load in order to stay below the yield point.

When each of the four panels had been tested once, the

cutouts were enlarged to a 4 in. diameter. If any strain gages were destroyed in the process, new ones were installed. After the tests at 4 in. diameter, the holes were increased to 6 in. and the final tests were made.

Since all gages were on an axis of symmetry in a tangential orientation, with the exception of a few on the riveted doubler, the strain readings were converted to stresses by the formula;

$$\sigma_y = \epsilon_y E \frac{(2.05)}{(G.F.)}$$

The gage factor correction, (2.05)/(G.F.), was necessary because the Anderson Strainmeter is constructed with a built in gage factor allowance of 2.05. The stresses thus obtained from the tests are recorded in Tables I to XII.

An additional correction was necessary for some of the gages on the riveted doubler. Because of interfering rivets, some gages had to be located off of axis of symmetry. The method of correction for this effect is contained in Appendix A.

ANALYSIS OF RESULTS

As brought out in the previous section, it was impossible to obtain a condition of uniform tension in the panels. Since all gages on the front sheet of the panel were not duplicated on the rear, it was necessary to consider the gages on the front panel only. As indicated by gages 1 to 6, the front panel took from 48.4 to 52 percent of the total load. Table XIII gives the values of load taken by the front face for each run.

The results of all experimental runs were plotted and are illustrated as Fig.(11) to (22). The curves represent tangential stress, σ_θ, at the minimum section of the plate vs. radius from the center of the hole.

For the theoretical results, it was assumed that the balsa core took no part of the load. This amounts to an error of less than 1.5 percent; see App. B. The theory, as given in Ref.(c), for plates of non-infinite width was used for comparison with the panels having no doublers. The theoretical stress distributions, as given in that reference, were plotted on Figs. (11), (15), and (19). Examination of these plots show excellent agreement in the area of the cutout. The theoretical stress becomes higher than the measured stress for positions away from the cutout. This could be due to the non-uniform

condition of applied stress in the panel. All test curves are consistent in shape with the theoretical curves.

For the panels with doublers, the theory of Ref.(b) was used for comparison. The theoretical stress curves are shown on the remaining Figs. (11) to (22).

The test results for the bonded doublers show excellent correlation with the theory of Ref.(b). The most noticeable departure from the theory is for the .032 in. doubler with 5 in. cutout where the measured maximum stresses are higher than the predicted stresses; see Fig.(20). This is most probably due to the condition of non-infinite width of the test panels. As shown on Fig.(19), the stress for a 5 in. hole with no reinforcing is 2.24S instead of 3S for an infinite plate. The other departure from the theoretical curves occurs at the outer boundary of the doubler. This was expected, since the theory requires a discontinuity in the stress curve which could not possibly exist.

The maximum stresses at the cutout for the bonded doublers were taken from the respective plots and divided by the applied stress for all loadings. The values thus obtained were averaged and recorded in Table XIV. These same values were plotted on Fig.(3) which was developed in a previous section. The experimental points show excellent correlation with the theory developed for Fig.(3) when it is remembered that the theoretical curves are for a plate of infinite width.

It should also be mentioned that while the stresses at the holes were taken almost to the yield point of the material, which is 60,000 psi., no loss in transmission of stress by the Redux bonding was apparent.

The results obtained from the riveted external doubler are not consistent with the theory. It first appeared that the stress obtained at the cutout was less than the theory predicted. Since this was suspected to be false, an A-12 gage was installed on the inside plate for the run with the 6 in. hole. As shown on Fig.(22), the stress in the plate is considerably higher than the stress on the doubler and higher than the theoretical stress. It is obvious that the rivets were not effectively transmitting the load to the doubler. This may have been due to too few rivets, especially in the case of the 8 in. cutout, or due to the type rivets used. In that case the rivets were Huck Blind Rivets. The maximum measured stresses at the cutout boundary for the riveted panel are also recorded in Table XIV for comparison with the theoretical and bonded doubler stresses.

CONCLUSIONS

From the experimental results obtained in this test, it can be concluded that the chart, Fig.(3), developed for doubler plates for circular cutouts in an infinitely wide plane sheet may be useful in future design of doubler rings.

It can be further concluded that internal bonded doublers are superior to external riveted doublers in reducing stress concentration around a circular cutout in Metalite.

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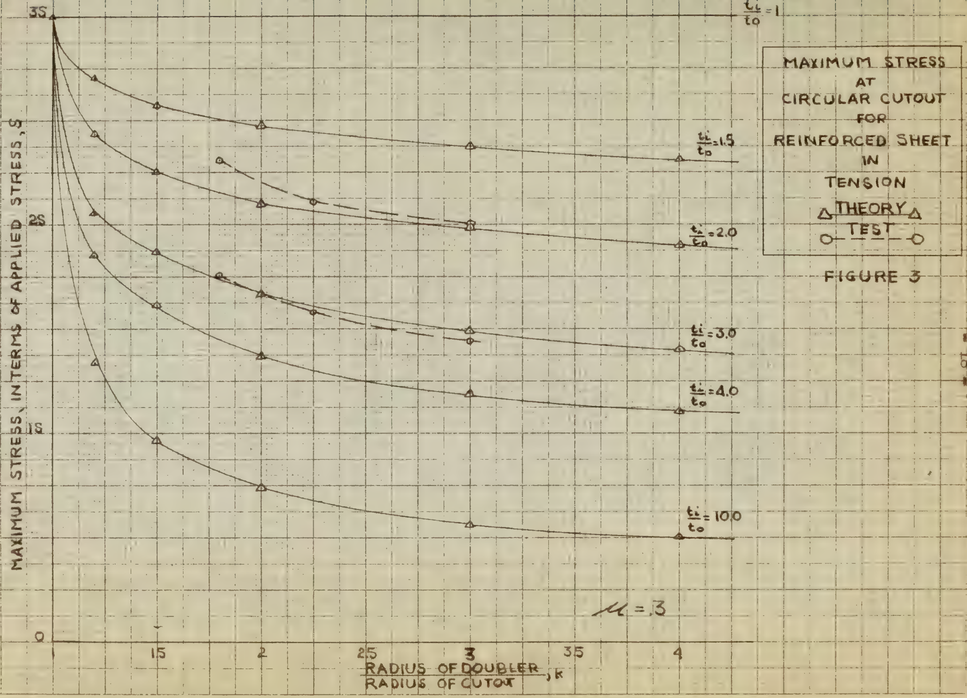
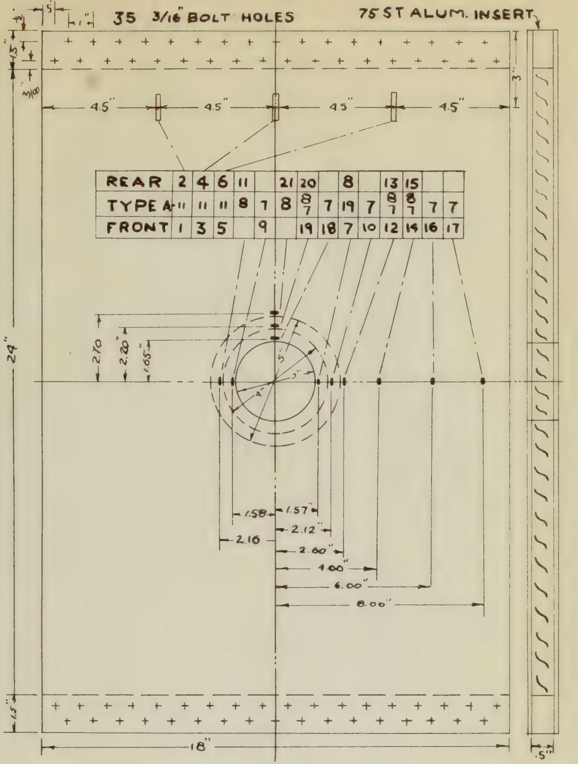
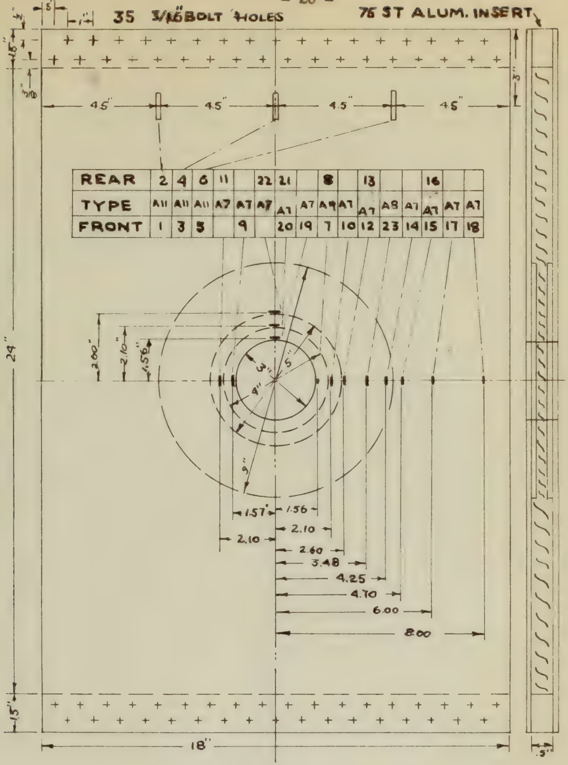

MAXIMUM STRESS AT CIRCULAR CUTOUT FOR REINFORCED SHEET IN TENSION
△ THEORY △
○ TEST ○
3S
2S
1S
0
1.5
2
2.5
3
3.5
4
MAXIMUM STRESS, INTERMS OF APPLIED STRESS, S
RADIUS OF DOUBLER / RADIUS OF CUTOUT, K
$\frac{t_i}{t_o} = 1$
$\frac{t_i}{t_o} = 1.5$
$\frac{t_i}{t_o} = 2.0$
$\frac{t_i}{t_o} = 3.0$
$\frac{t_i}{t_o} = 4.0$
$\frac{t_i}{t_o} = 10.0$
$\mu = .3$

FIGURE 3


35 3/16" BOLT HOLES
75 ST ALUM. INSERT
.5"
1"
.5"
3/100
4.5"
4.5"
4.5"
4.5"
3"
REAR 2 4 6 11 21 20 8 13 15
TYPE A 11 11 11 8 7 8 8/7 7 19 7 8/7 8/7 7 7
FRONT 1 3 5 9 19 18 7 10 12 14 16 17
2.70
2.20"
1.65"
5"
3"
4"
1.58
1.57"
2.16
2.12"
2.60"
4.00"
6.00"
8.00"
24"
1.5"
18"
.5"

METALITE PANEL
.032 in FACES, 75 ST6 ALCLAD
.5 in. CORE, 7-9#/ft³ BALSA
NO DOUBLER

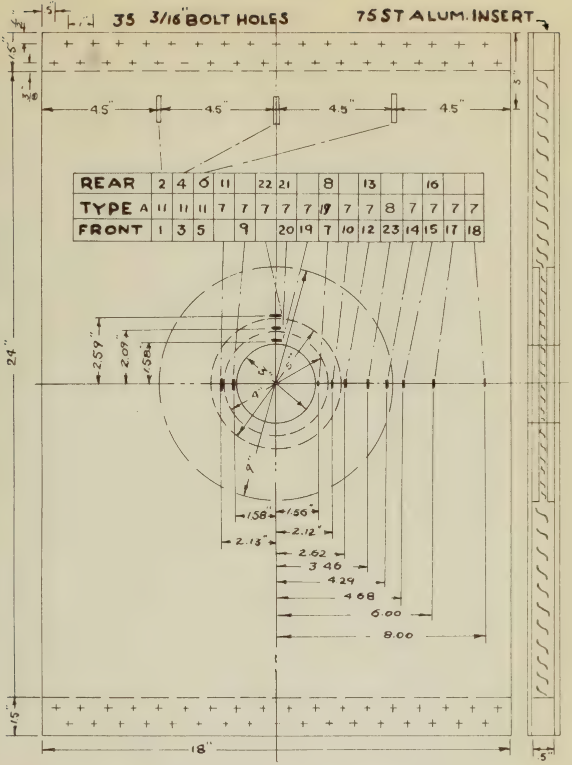
FIGURE 4


.5"
1"
35 3/16" BOLT HOLES
75 ST ALUM. INSERT
1.5"
3/10
3"
4.5"
4.5"
4.5"
4.5"
24"
3.00"
2.10"
1.56"
3"
4"
5"
9"
1.57"
1.56
2.10
2.10
2.60
3.48
4.25
4.70
6.00
8.00
1.5"
18"
.5"

REAR	2	4	6	11		22	21		8		13			16			
TYPE	A11	A11	A11	A7	A7	A7	A7	A7	A9	A7	A7	A8	A7	A7	A7	A7	
FRONT	1	3	5		9		20	19	7	10	12	23	14	15	17	18	

METALITE PANEL
.032 in FACES, 75 ST-6 ALCLAD
.5 in CORE, 7-9#/ft³ BALSA
.032 in DOUBLER, 75ST-6 ALCLAD

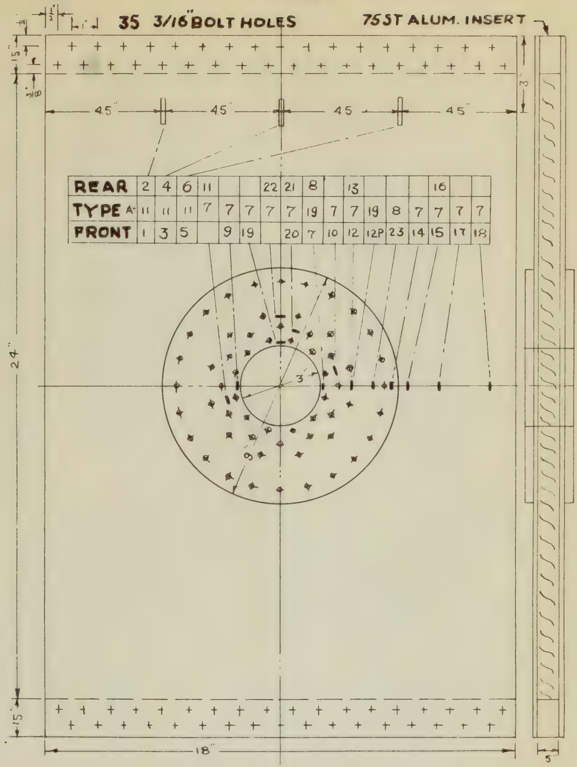
FIGURE 5


35 3/16" BOLT HOLES
75 ST ALUM. INSERT
.5"
1/4"
1.5"
3/8"
3"
4.5"
4.5"
4.5"
4.5"
24"
2.59"
2.09"
1.58"
3"
5"
A"
9"
1.58"
1.56"
2.12"
2.13"
2.62
3 46
4.29
4 68
6.00
8.00
1.5"
18"
.5"

REAR	2	4	6	11		22	21		8		13			16		
TYPE A	11	11	11	7	7	7	7	7	19	7	7	8	7	7	7	7
FRONT	1	3	5		9		20	19	7	10	12	23	14	15	17	18

METALITE PANEL
.032in. FACES, 75 ST-6 ALCLAD
.5in. CORE, 7-9#/ft³ BALSA
.064in. DOUBLER, 75ST6 ALCLAD

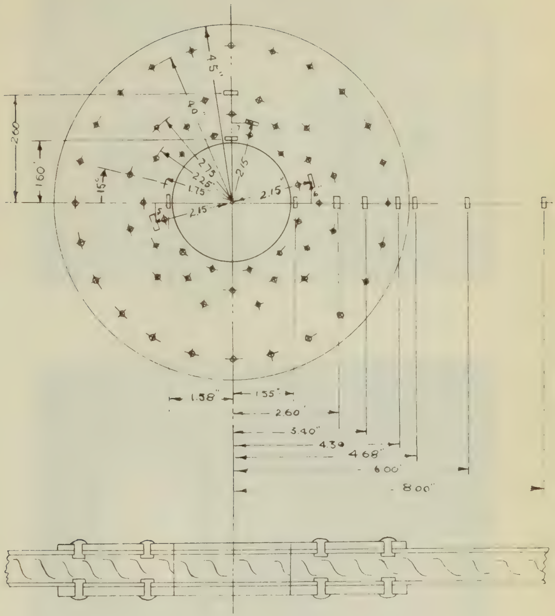
FIGURE 6


35 3/16" BOLT HOLES
75ST ALUM. INSERT
1/2"
1"
1/2"
1.5"
3/8"
3"
4.5"
4.5"
4.5"
4.5"
3"
9"
24"
1.5"
18"
.5"

REAR	2	4	6	11			22	21	8		13				16		
TYPE A-	11	11	11	7	7	7	7	7	19	7	7	19	8	7	7	7	7
FRONT	1	3	5		9	19		20	7	10	12	12P	23	14	15	17	18

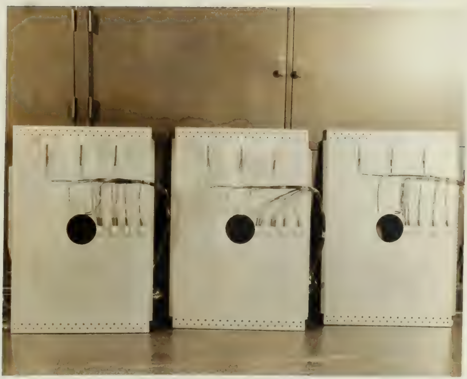
METALITE PANEL
.032 in FACES, 75ST-6 ALCLAD
.5 in CORE, 7-9#/Ft³ BALSA
.064 in RIVETED DOUBLER

FIGURE 7


4.5"
4.0"
2.60
1.60"
15°
2.75
2.25
1.75"
2.15
2.15"
2.15
7
6
5
1.58"
1.55"
2.60'
3.40"
4.30
4.68"
6.00'
8.00"

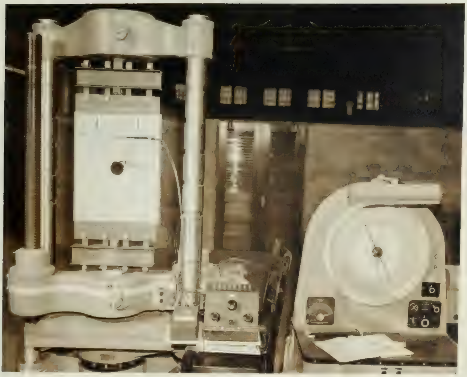
RIVETED DOUBLER
HUCK BRAZIER HEAD
BLIND RIVETS
1/8" DIAMETER

FIGURE 7a



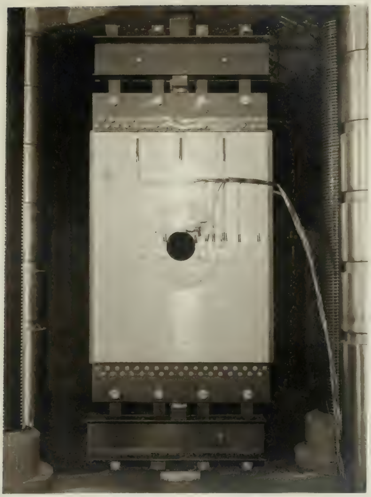
Metalite Test Panels

Figure 8



Test Setup

Figure 9



Test Panel With Attaching Apparatus

Figure 10

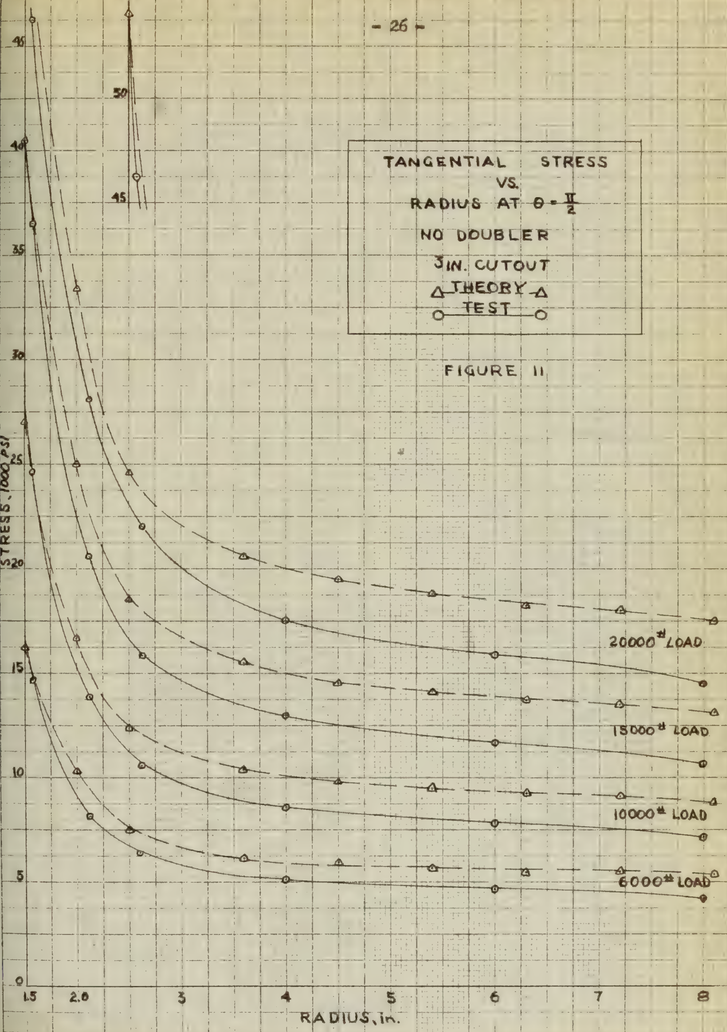

TANGENTIAL STRESS
VS.
RADIUS AT θ = π/2
NO DOUBLER
3 IN. CUTOUT
Δ THEORY Δ
○ TEST ○
STRESS, 1000 PSI
RADIUS, in.
0
5
10
15
20
25
30
35
40
45
45
50
1.5
2.0
3
4
5
6
7
8
20000# LOAD
15000# LOAD
10000# LOAD
6000# LOAD

FIGURE 11

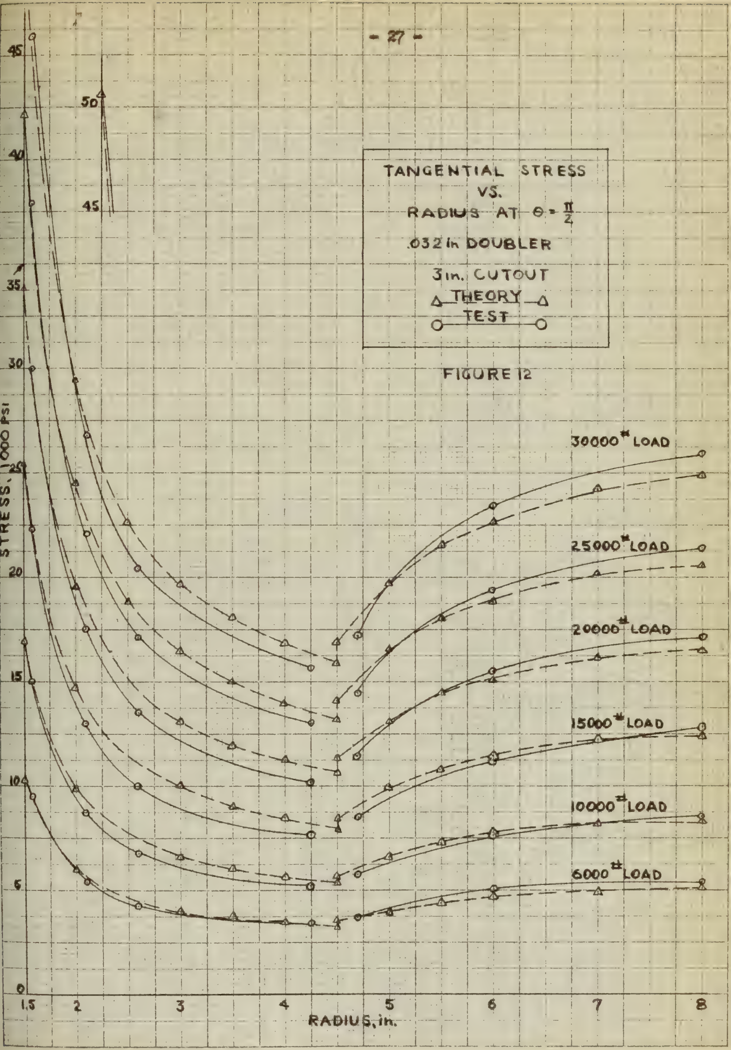

TANGENTIAL STRESS
VS.
RADIUS AT θ = π/2
.032 in DOUBLER
3 in. CUTOUT
THEORY
TEST
30000# LOAD
25000# LOAD
20000# LOAD
15000# LOAD
10000# LOAD
6000# LOAD
STRESS, 1000 PSI
RADIUS, in.
0
5
10
15
20
25
30
35
40
45
45
50
1.5
2
3
4
5
6
7
8

FIGURE 12

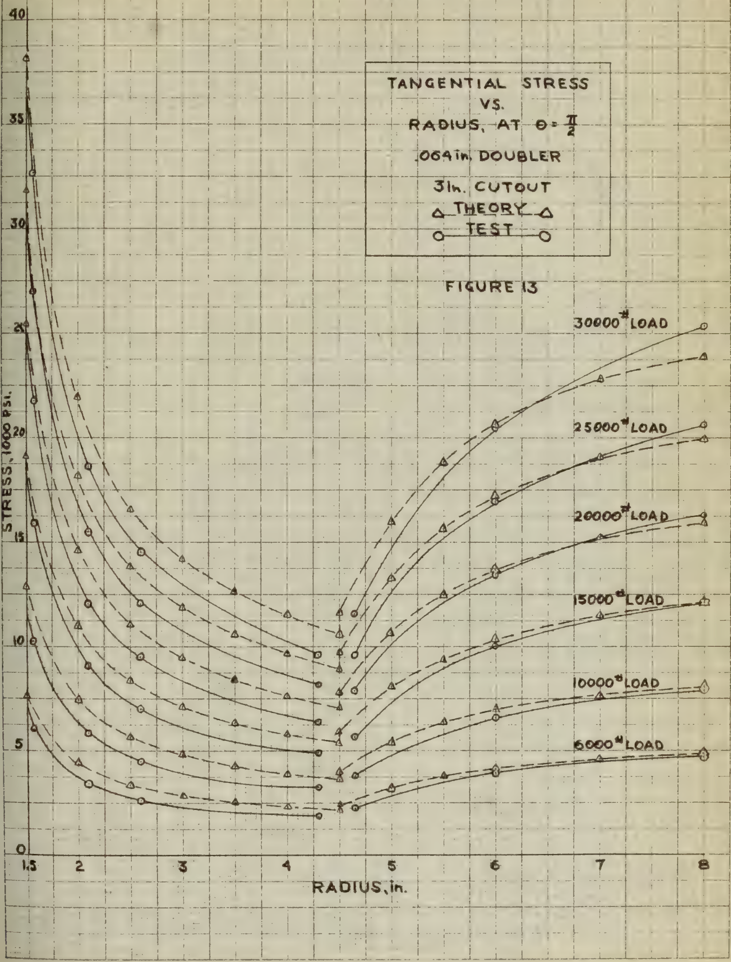

TANGENTIAL STRESS
VS.
RADIUS, AT $\theta = \frac{\pi}{2}$
.064 in. DOUBLER
3 In. CUTOUT
△ THEORY △
○ TEST ○
30000# LOAD
25000# LOAD
20000# LOAD
15000# LOAD
10000# LOAD
6000# LOAD
STRESS, 1000 P.S.I.
40
35
30
25
20
15
10
5
0
1.5
2
3
4
5
6
7
8
RADIUS, in.

FIGURE 13

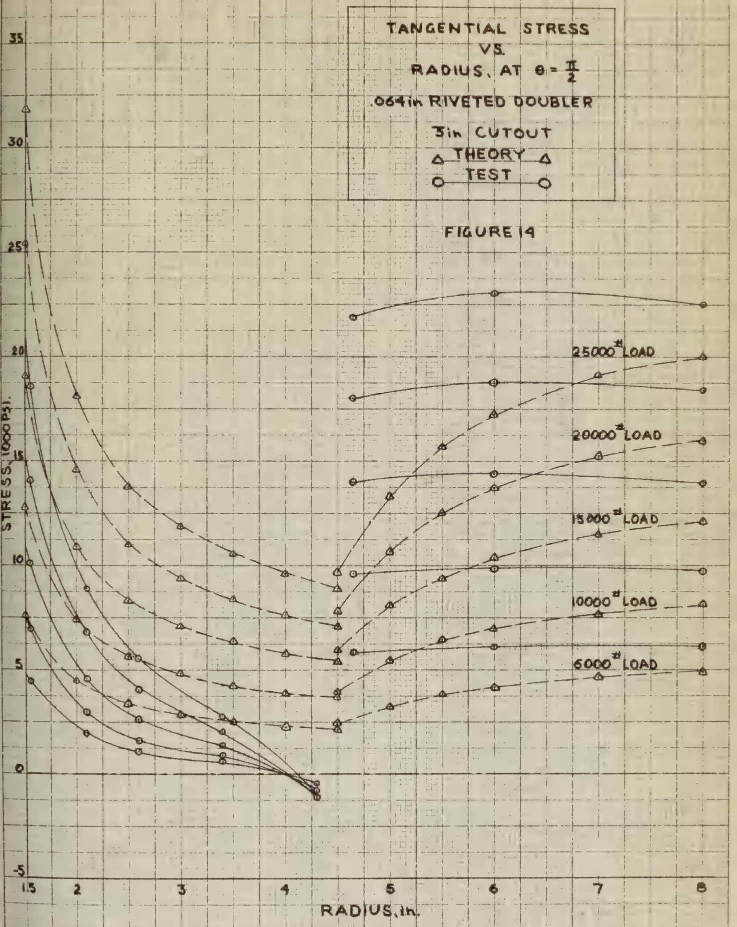

TANGENTIAL STRESS
VS.
RADIUS, AT $\theta = \frac{\pi}{2}$
.064in RIVETED DOUBLER
3in CUTOUT
△ THEORY △
○ TEST ○
25000# LOAD
20000# LOAD
15000# LOAD
10000# LOAD
6000# LOAD
STRESS, (1000 PSI).
35
30
25
20
15
10
5
0
-5
1.5
2
3
4
5
6
7
8
RADIUS, in.

FIGURE 14

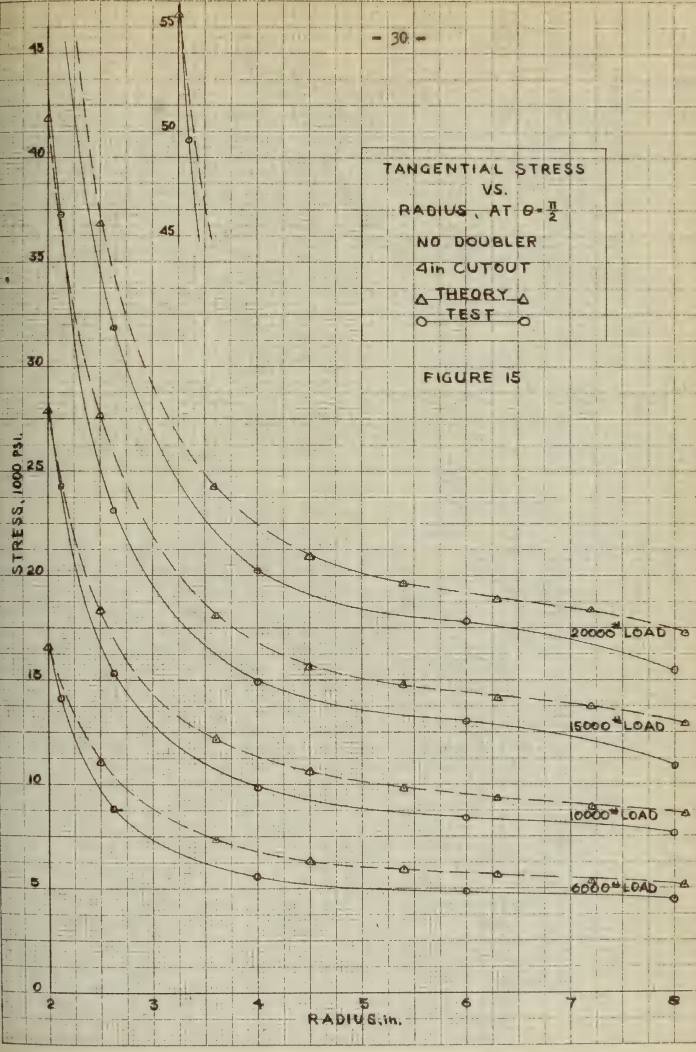

TANGENTIAL STRESS
VS.
RADIUS, AT $\theta = \frac{\pi}{2}$
NO DOUBLER
4in CUTOUT
THEORY
TEST
STRESS, 1000 PSI.
RADIUS, in.
0
5
10
15
20
25
30
35
40
45
50
55
2
3
4
5
6
7
8
20000# LOAD
15000# LOAD
10000# LOAD
6000# LOAD

FIGURE 15

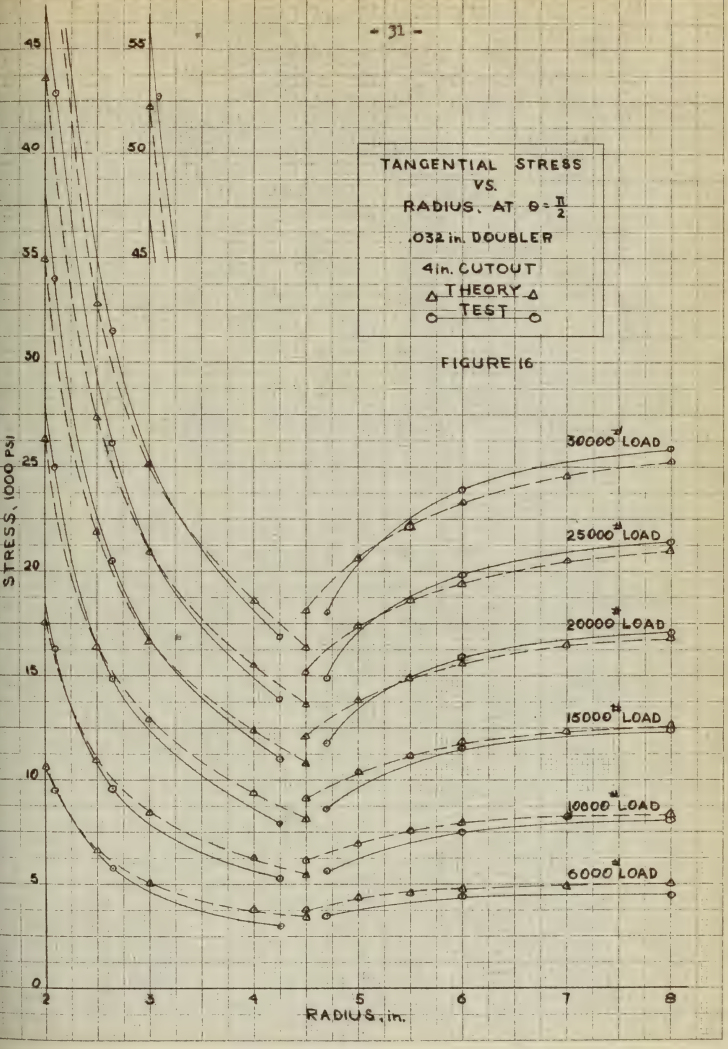

TANGENTIAL STRESS
VS.
RADIUS, AT $\theta = \frac{\pi}{2}$
.032 in. DOUBLER
4 in. CUTOUT
THEORY
TEST
30000# LOAD
25000# LOAD
20000# LOAD
15000# LOAD
10000# LOAD
6000# LOAD
STRESS, 1000 PSI
0
5
10
15
20
25
30
35
40
45
45
50
55
RADIUS, in.
2
3
4
5
6
7
8

FIGURE 16

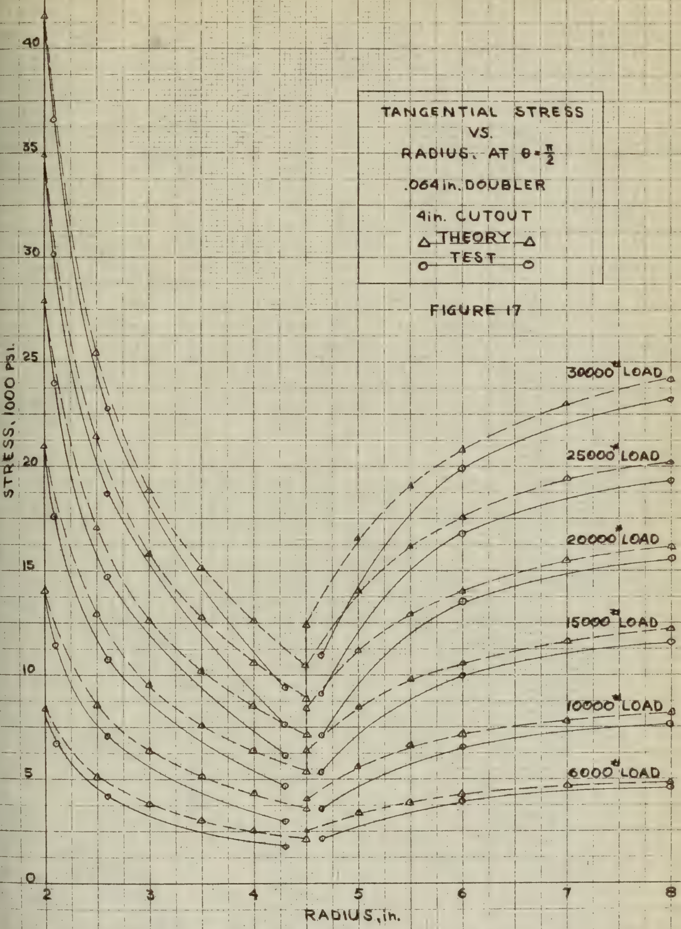

TANGENTIAL STRESS
VS.
RADIUS, AT $\theta = \frac{\pi}{2}$
.064 in. DOUBLER
4 in. CUTOUT
THEORY
TEST
30000# LOAD
25000# LOAD
20000# LOAD
15000# LOAD
10000# LOAD
6000# LOAD
STRESS, 1000 PSI.
0
5
10
15
20
25
30
35
40
RADIUS, in.
2
3
4
5
6
7
8

FIGURE 17

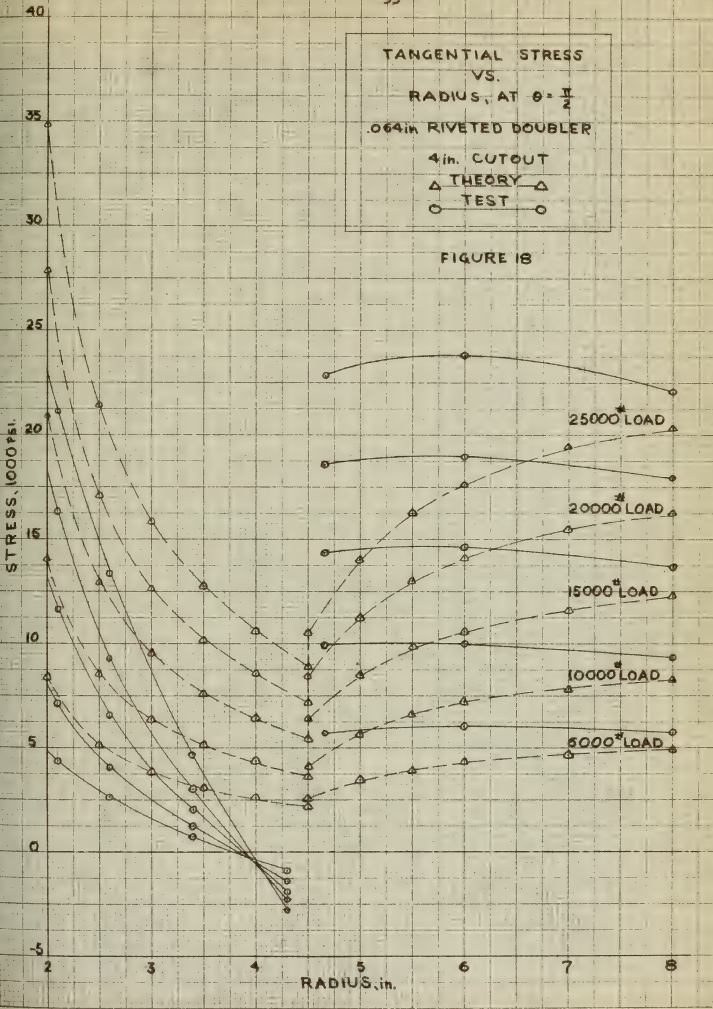

TANGENTIAL STRESS
VS.
RADIUS, AT $\theta = \frac{\pi}{2}$
.064in. RIVETED DOUBLER
4in. CUTOUT
THEORY
TEST
25000# LOAD
20000# LOAD
15000# LOAD
10000# LOAD
5000# LOAD
STRESS, 1000 PSI.
RADIUS, in.
40
35
30
25
20
15
10
5
0
-5
2
3
4
5
6
7
8

FIGURE 18

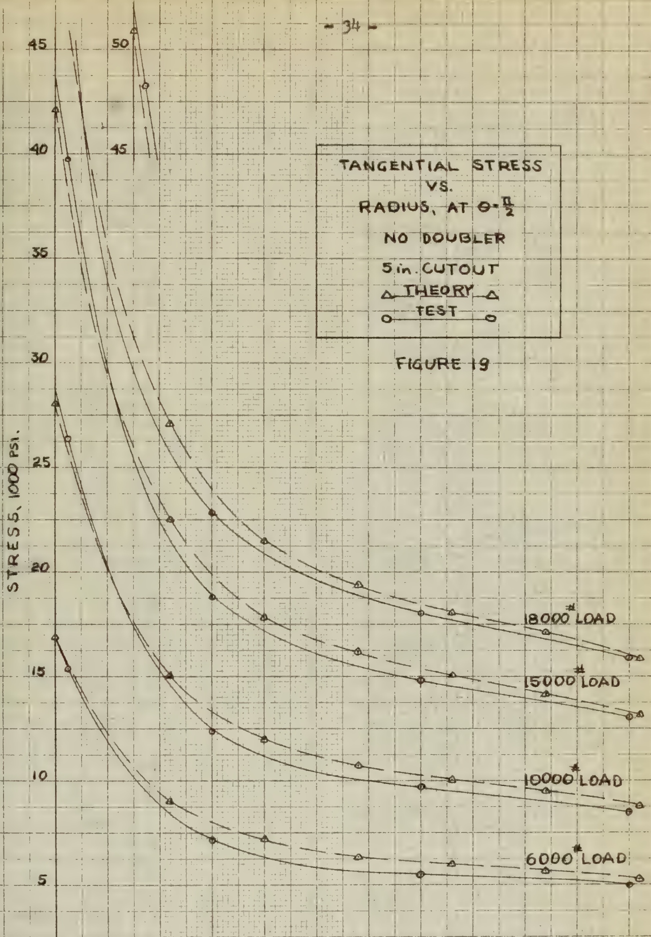

TANGENTIAL STRESS
VS.
RADIUS, AT $\theta = \frac{\pi}{2}$
NO DOUBLER
5 in. CUTOUT
THEORY
TEST
18000# LOAD
15000# LOAD
10000# LOAD
6000# LOAD
STRESS, 1000 PSI.
RADIUS, in.
0
5
10
15
20
25
30
35
40
45
45
50
2.5
3
4
5
6
7
8

FIGURE 19

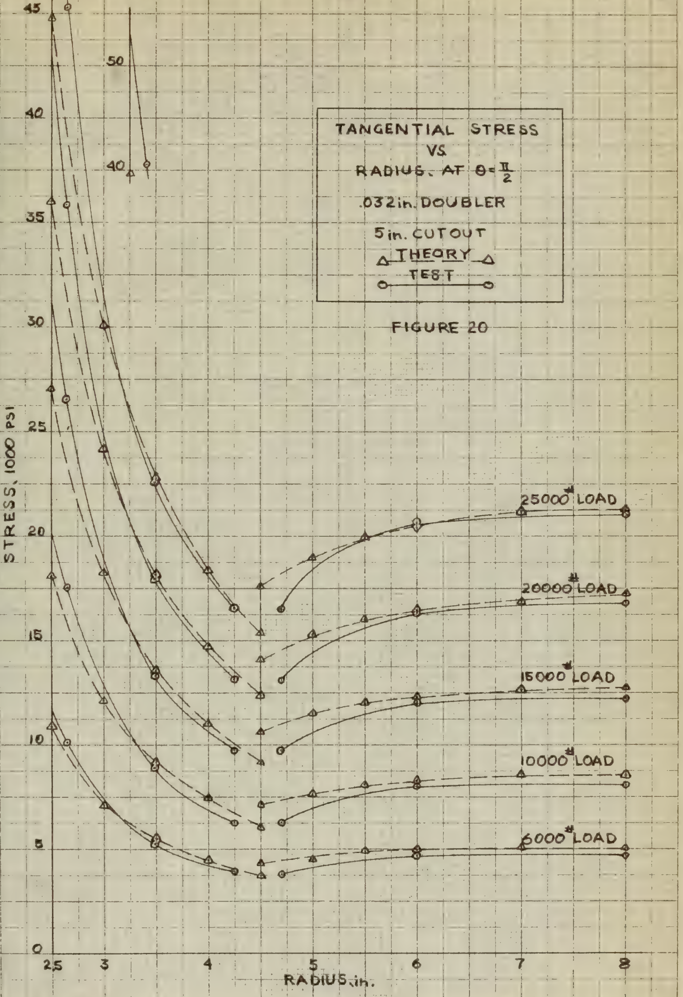

TANGENTIAL STRESS
VS
RADIUS. AT $\theta = \frac{\pi}{2}$
.032 in. DOUBLER
5 in. CUTOUT
THEORY
TEST
STRESS, 1000 PSI
RADIUS, in.
0
5
10
15
20
25
30
35
40
45
50
40
2.5
3
4
5
6
7
8
25000# LOAD
20000# LOAD
15000# LOAD
10000# LOAD
5000# LOAD

FIGURE 20

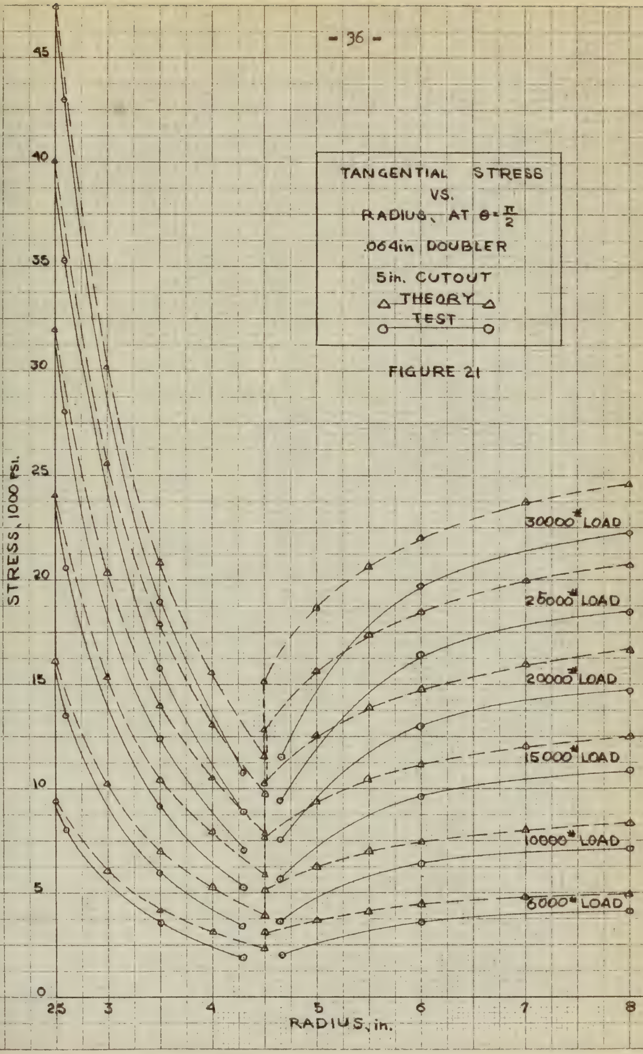

TANGENTIAL STRESS
VS.
RADIUS, AT $\theta = \frac{\pi}{2}$
.064in DOUBLER
5in. CUTOUT
△ THEORY △
○ TEST ○
STRESS, 1000 PSI.
RADIUS, in.
0
5
10
15
20
25
30
35
40
45
2.5
3
4
5
6
7
8
30000# LOAD
25000# LOAD
20000# LOAD
15000# LOAD
10000# LOAD
5000# LOAD

FIGURE 21

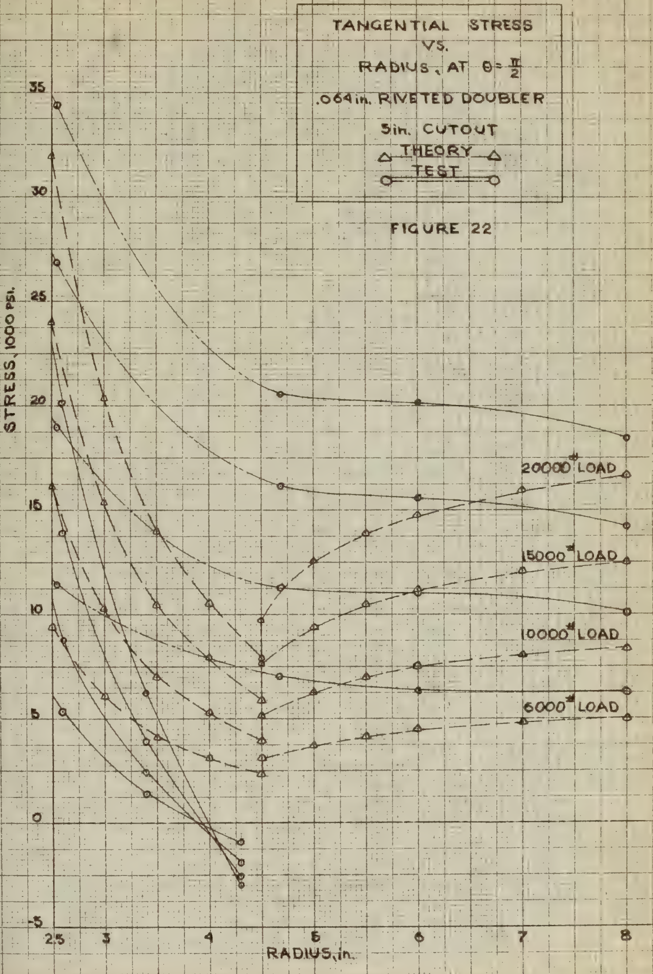

TANGENTIAL STRESS
VS.
RADIUS, AT θ = π/2
.064 in. RIVETED DOUBLER
5 in. CUTOUT
THEORY
TEST
STRESS, 1000 PSI.
35
30
25
20
15
10
5
0
-5
RADIUS, in.
2.5
3
4
5
6
7
8
20000# LOAD
15000# LOAD
10000# LOAD
6000# LOAD

FIGURE 22

TABLE I

STRESSES FROM STRAIN GAGE READINGS

METALITE PANEL - NO DOUBLER - 3in. CUTOUT

Gage	Type	Gage Fact	Load, lbs.	100	6100	10100	15100	20100
					psi	psi	psi	psi
1	A-11	2.08		0	5990	10050	14850	19550
2	↓	↓		↓	5370	9620	14650	19600
3	↓	↓		↓	5020	9620	13000	17250
4	↓	↓		↓	5720	8620	13300	17800
5	↓	↓		↓	5990	9510	13850	18250
6	↓	↓		↓	6090	10150	14950	19650
7	A-19	1.63		↓	13880	23000	34200	45400
8	↓	↓		↓	14920	25300	37200	49200
9	A-7	1.91		↓	15380	25700	38900	48900
10	↓	↓		↓	8230	13800	20650	28200
11	A-8	1.70		↓	9190	16150	23750	32000
12	A-7	1.91		↓	6300	10650	15900	22050
13	A-8	1.70		↓	6700	11950	17900	23900
14	A-7	1.91		↓	5090	8670	13000	17550
15	A-8	1.70		↓	5590	10000	14800	19850
16	A-7	1.91		↓	4640	7840	11700	15900
17	↓	↓		↓	4200	7130	10700	14500
18	↓	↓		↓	-5090	-8950	-13300	-17550
19	↓	↓		↓	-1328	-2430	-3535	-4760
20	A-8	1.70		↓	-1105	-1990	-2930	-4350
21	↓	↓		↓	-884	-1490	-2210	-3290

TABLE II

STRESSES FROM STRAIN GAGE READINGS

METALITE PANEL -.032 in. DOUBLER - 3 in. CUTOUT

Gage	Type	Gage Fact.	Load, lbs.						
			100	6100	10100	15100	20100	25100	30100
				psi	psi	psi	psi	psi	psi
1	A-11	2.08	0	6290	10100	14500	18550	22500	26000
2				5240	9040	13900	18550	23500	28100
3				6440	9950	14550	18790	23300	27900
4				5880	9950	15210	20400	25900	31000
5				5450	8120	12490	17150	21600	26400
6	↓	↓		5940	8920	14100	19600	25100	30750
7	A-19	1.63		9050	14380	21200	28500	35900	43200
8	↓	↓		9500	16250	24100	31800	40700	48600
9	A-7	1.91		9600	15700	23200	31300	39800	48500
10				5470	8730	12950	17580	22100	26750
11				5640	9600	14450	19320	24300	29300
12				4200	6740	9940	13500	17110	20450
13				4420	7730	11700	15500	19600	23200
14				3240	5200	7620	10200	12920	15530
15				3650	5750	8400	11400	14350	17130
16				3870	6420	9500	12800	12800	19000
17				4860	7620	11300	15480	19300	23350
18				5260	8510	12760	17150	21400	25850
19				-3760	-6070	-8900	-11800	-14350	-17700
20				-1048	-1765	-2540	-3310	-4080	-4860
21				-940	-1655	-2430	-3310	-4140	-5080
22	↓	↓	↓	-552	-906	-1545	-1880	-2320	-2870

TABLE III

STRESSES FROM STRAIN GAGE READINGS

METALITE PANEL - .064 in. DOUBLER - 3 in. CUTOUT

Gage	Type	Gage Fact.	Load, lbs.						
			100	6100	10100	15100	20100	25100	30100
				psi	psi	psi	psi	psi	psi
1	A-11	2.08	0	6450	9940	14600	19300	24200	28650
3	↓	↓	↓	6240	10050	15150	20190	25350	30400
4	↓	↓	↓	5120	8710	13550	18250	23250	28300
5	↓	↓	↓	5270	9230	14400	19280	24450	29450
6	↓	↓	↓	5020	8160	13000	17450	22500	27200
7	A-19	1.63	↓	6020	10180	15500	20950	26550	32100
8	↓	↓	↓	7180	11720	17620	22950	28750	34400
9	A-7	1.91	↓	6180	10450	16100	21550	27450	33300
10	↓	↓	↓	3420	5850	9060	12100	15450	18680
11	↓	↓	↓	4320	6940	10100	13400	16650	19900
12	↓	↓	↓	2650	4475	6960	9440	12050	14500
13	↓	↓	↓	3430	5640	8280	10910	13700	16180
14	↓	↓	↓	1768	3200	4860	6460	8060	9600
15	↓	↓	↓	2210	3755	5690	7780	9600	11500
16	↓	↓	↓	2780	4580	6630	8840	10850	12900
17	↓	↓	↓	3860	6520	10000	13480	16950	20500
19	↓	↓	↓	4640	7900	12050	16450	20700	25350
20	↓	↓	↓	-2210	-3750	-5910	-8060	-10280	-12350
21	↓	↓	↓	-663	-992	-1490	-1985	-2485	-3040
22	↓	↓	↓	-442	-607	-994	-1150	-1435	-6070

TABLE IV

STRESSES FROM STRAIN GAGE READINGS

METALITE PANEL - .064 in. RIVETED DOUBLER - 3 in. CUTOUT

Gage	Type	Gage Fact.	Load, lbs.					
			100	6100	10100	15100	20100	25100
				psi	psi	psi	psi	psi
1	A-11	2.08	0	4160	6800	10050	13600	17000
2				4470	7610	11900	16450	20700
3				5520	8520	12340	16450	22000
4				4820	7820	12900	17250	22450
5				5130	8260	12090	16200	19900
6				4260	7560	12400	17200	22200
7	A-19	1.63		4140	6600	9700	13850	18400
8				3495	5570	9040	13270	18200
9	A-7	1.91		4750	7240	10390	14350	18690
10				1600	2710	4150	6080	8020
11				1270	2210	3650	5520	7680
12				1050	1660	2710	4090	5530
13				884	1440	2375	3780	5310
14				-498	-775	-1085	-1105	-1105
15				5820	9520	13950	18150	21950
16				4970	8560	12850	16850	20750
17				6090	9800	14400	18800	23150
18				6090	9400	13950	18350	22450
19				-2710	-4360	-6580	-8950	-11480
20				-445	-608	-940	-1380	-1660
21				-165	0	221	331	387
22				0	0	442	718	829
23	A-8	1.79		648	943	1415	2065	2770

TABLE V

STRESSES FROM STRAIN GAGE READINGS

METALITE PANEL - NO DOUBLER - 4 in. CUTOUT

Gage	Type	Gage Fact.	Load, lbs. 0	6100	10100	15100	20100
				psi	psi	psi	psi
1	A-11	2.08	0	5420	9080	13850	18560
2	↓	↓	↓	6350	10100	14610	19230
3	↓	↓	↓	4320	7610	12970	16380
4	↓	↓	↓	5270	8520	12630	16720
5	↓	↓	↓	5020	8830	13440	18160
6	↓	↓	↓	5480	9130	13540	18050
10	A-7	1.91	↓	14150	24250	37200	49600
11	A-8	1.79	↓	16250	26100	38300	51300
12	A-7	1.91	↓	8790	15260	23200	31620
13	A-8	1.79	↓	10250	16640	23700	32700
14	A-7	1.91	↓	5530	9780	14930	20250
15	A-8	1.79	↓	6950	11080	16300	21500
16	A-7	1.91	↓	4760	8350	13000	17700
17	↓	↓	↓	4320	7575	10830	15490
20	A-8	1.79	↓	-4480	-7615	-11320	-15100
21	↓	↓	↓	-1711	-2950	-4370	-5725

TABLE VI

STRESSES FROM STRAIN GAGE READINGS

METALITE PANEL - .032 in. DOUBLER - 4 in. CUTOUT

Gage	Type	Gage Fact.	Load, lbs.						
			100	6100	10100	15100	20100	25100	30100
				psi	psi	psi	psi	psi	psi
1	A-11	2.08	0	5500	9350	13860	18350	22850	27500
2				5840	9850	14520	19200	23950	28850
3				5130	8700	13200	17720	22250	27100
4				5640	9550	14320	19250	23750	28835
5				5790	9300	13860	18500	23350	28050
6				5930	9700	14720	19700	24500	28150
10	A-7	1.91		9550	16300	25000	34000	42900	52700
11				11400	18600	27600	37200	46300	56200
12				5750	9550	14910	20450	14870	31500
13				7300	11820	17310	23200	27850	35150
14				2985	5250	7960	10920	13850	16800
15				3315	5640	8620	11750	14920	18000
16				4420	6950	10100	13420	16520	19650
17				4420	7520	11500	15750	19800	23900
18				4750	8120	12500	17010	21400	25800
20				-3210	-5530	-8180	-10720	-13600	-16400
21				-3210	-5360	-8280	-11100	-13800	-16750
22				-1380	-2320	-3430	-4420	-5530	-6580

TABLE VII

STRESSES FROM STRAIN GAGE READINGS

METALITE PANEL - .064 in. DOUBLER - 4 in. CUTOUT

Gage	Type	Gage Fact.	Load, lbs.						
			100	6100	10100	15100	20100	25100	30100
				psi	psi	psi	psi	psi	psi
1	A-11	2.08	0	5580	9590	14710	19640	24850	29500
3	↓	↓	↓	5220	8830	13700	18500	23550	28400
4	↓	↓	↓	4670	7920	12400	17150	22100	26800
5	↓	↓	↓	6590	10650	15820	20300	25850	30850
6	↓	↓	↓	5220	8620	13200	17730	21800	26100
10	A-7	1.91	↓	6740	11450	17600	23900	30200	36650
11	↓	↓	↓	7960	13160	19250	24900	27500	39150
12	↓	↓	↓	4200	7080	10750	14700	18680	22750
13	↓	↓	↓	5200	8300	12380	16480	20450	24800
14	↓	↓	↓	1770	2990	4700	6190	7740	9400
15	↓	↓	↓	2100	3540	5310	7190	9060	10920
16	↓	↓	↓	2765	4530	6410	8520	10380	12430
17	↓	↓	↓	3930	6520	10000	13600	16750	19900
18	↓	↓	↓	4640	7630	11600	15500	19350	23200
20	↓	↓	↓	-2710	-4580	-7190	-9840	-12600	-15480
21	↓	↓	↓	-2700	-4530	-6740	-8950	-11380	-13820
22	↓	↓	↓	-995	-1715	-2355	-3320	-4150	-5090

TABLE VIII

STRESSES FROM STRAIN GAGE READINGS

METALITE PANEL - .064 in. RIVETED DOUBLER - 4 in. CUTOUT

Gage	Type	Gage Fact.	Load, lbs.					
			100	6100	10100	15100	20100	25100
				psi	psi	psi	psi	psi
1	A-11	2.08	0	4460	7100	10750	14450	18000
2				4730	8120	12700	17250	21800
3				6190	9660	14000	18450	22500
4				5630	9340	14320	18950	23850
5				5020	8020	12190	16150	20300
6				4480	8020	12750	17150	21700
10	A-7	1.91		3870	6420	10320	14600	19000
11				3870	5860	9000	12650	17800
12				2655	4090	6520	9280	16050
13				2655	4150	6640	6640	13380
14				-884	-1382	-1980	-2320	-2765
15				5970	9890	14380	18620	22850
16				4860	8560	12820	17010	21100
17				6080	9840	14640	18950	23600
18				5750	9280	13700	17900	22200
20				-2210	-3540	-5470	-7410	-9620
22				-775	-775	-885	-1490	-2360
23	A-8	1.79		825	1300	2005	2950	4720

TABLE IX

STRESSES FROM STRAIN GAGE READINGS

METALITE PANEL - NO DOUBLER - 5 in. CUTOUT

		Gage	Load, lbs.				
Gage	Type	Fact.	100	6100	10100	15100	18100
				psi	psi	psi	psi
1	A-11	2.08	0	5420	9120	13800	16540
2				5830	9520	14300	17100
3				5120	8660	13195	15780
4				5320	8970	13395	15930
5				5830	9730	14700	17430
6				5840	9730	14600	17430
12	A-7	1.91		15350	26300	39800	48200
13	A-8	1.79		16750	27500	41500	49500
14	A-7	1.91		7170	12360	18760	22800
15	A-8	1.79		8300	13740	20100	24150
16	A-7	1.91		5575	9660	14850	18000
17	A-7	1.91		4690	8540	13030	15800
21	A-8	1.79		-5300	-8720	-12950	-15730

TABLE X

STRESSES FROM STRAIN GAGE READINGS

METALITE PANEL - .032 in. DOUBLER - 5 in. CUTOUT

Gage	Type	Gage Fact.	Load, lbs.					
			100	6100	10100	15100	20100	25100
				psi	psi	psi	psi	psi
1	A-11	2.08	0	6200	10000	14630	19100	23750
2				6050	10050	14950	19650	24650
3				5030	8130	12300	16450	20750
4				4780	8240	12600	16870	21350
5				4780	8390	13000	17620	22500
6				4880	8640	13500	18400	23200
12	A-7	1.91		10150	17480	26550	35800	45300
13				12500	20550	30400	40500	50600
14				3870	6360	9720	13150	16650
15				3760	6300	9770	13150	16500
16				4750	7740	11150	14910	18300
17				4640	8010	12050	16400	20550
18				4640	8180	12380	16800	21050
22				5250	8860	13300	17950	22500
23	A-8	1.81		-4420	-7010	-10480	-14140	-17690

TABLE XI

STRESSES FROM STRAIN GAGE READINGS

METALITE PANEL - .064 in. DOUBLER - 5 in. CUTOUT

Gage	Type	Gage Fact.	Load, lbs.						
			100	6100	10100	15100	20100	25100	30100
				psi	psi	psi	psi	psi	psi
1	A-11	2.08	0	5230	9200	14320	19100	24150	28750
3				4920	8530	13500	17930	22600	27100
4				5380	8950	13500	18050	23500	27300
5				6100	10050	14940	19550	24500	28900
6	↓	↓		5690	9200	13560	17880	22200	26700
12	A-7	1.91		8000	13600	20550	28100	35300	43000
13				9380	15475	11600	30200	37900	45700
14				1935	3370	5250	7010	8830	10750
15				2100	3650	5640	7510	9440	11450
16				2875	4590	6740	8610	10820	12850
17				3645	6300	9550	12930	16350	19700
18				4080	7130	10870	14650	18400	22200
22	↓	↓		-3650	-6075	-9060	-11950	-15200	-18400
23	A-8	1.81	↓	3500	6060	9210	12360	15730	18900

TABLE XII

STRESSES FROM STRAIN GAGE READINGS

METALITE PANEL - .064 in. RIVETED DOUBLER - 5 in. CUTOUT

Gage	Type	Gage Fact.	Load, lbs.				
			100	6100	10100	15100	20100
				psi	psi	psi	psi
1	A-11	2.08	0	4270	7110	10580	14520
2				4675	8030	12200	16550
3				5180	8220	12300	16350
4				4970	8120	12300	16550
5				4570	7760	11700	15950
6				4370	7820	12100	16450
12p	A-19	1.63		11420	18900	26850	34400
12	A-7	1.91		5300	8770	13950	20200
13				5135	7840	12480	18050
14				-992	-1990	-2595	-2980
15				6960	11050	16150	20600
16				5575	9830	14400	18950
17				6570	11050	15630	20150
18				6290	10000	14240	18550
22	A-8	1.81		-2450	-4370	-7110	-10310
23				1400	2450	3970	6240

TABLE XIII

PERCENT OF APPLIED LOAD

TAKEN BY FRONT FACE OF TEST PANELS

Panel	Percent of load taken by front face		
	3 in. hole	4 in.hole	5 in. hole
No Doubler	49.90	47.80	49.20
.032 Doubler	49.02	48.60	49.60
.064 Doubler	52.60	53.00	50.50
Rivet Doubler	49.10	48.40	49.10

TABLE XIV

MAXIMUM STRESS AT CUTOUT

Panel	3 in. Hole		4 in. Hole		5 in. Hole	
	Test	Theor.	Test	Theor.	Test	Theor.
No Doubler	3.170	3.200	3.215	3.105	3.31	3.237
.032 Doubler	1.975	1.950	2.11	2.06	2.33	2.15
.064 Doubler	1.433	1.480	1.575	1.600	1.75	1.730
Rivet Doubler	.90	1.480	1.02	1.600	1.19	1.730
Rivet Do.(face)					2.14	1.730

APPENDIX A

STRESS FORMULAS AS DERIVED IN REF.(b).

The following symbols were used in Ref.(b):

S - applied stress.

θ - angle measured from the direction of the load.

a - radius of cutout.

b - external radius of reinforcing ring.

ti - thickness of ring and sheet.

to - thickness of sheet.

μ - Poisson's ratio.

E - modulus of elasticity.

$\sigma_{\theta i}$ - tangential stress inside of the ring.

$\sigma_{\theta o}$ - tangential stress outside of the ring.

σ_{ri} - radial stress inside of the ring.

σ_{ro} - radial stress outside of the ring.

The solution, as developed in Ref.(b), for a circular hole with a reinforcement ring considers the stresses in the area of the cutout as being created by two forces existing at a radius far away from the cutout; the radial force $\frac{1}{2}S$, and the combination of the radial force, $\frac{1}{2}S \operatorname{Cos} 2\theta$, and the shearing force, $-\frac{1}{2}S \operatorname{Sin} 2\theta$.

Stresses due to the radial component ($\frac{1}{2}S$):

$$\sigma_{ri} = E\left[(1+\mu)A_i - (1-\mu)\frac{B_i}{r^2}\right]; \quad \sigma_{\theta i} = E\left[(1+\mu)A_i + (1-\mu)\frac{B_i}{r^2}\right]$$

$$\sigma_{ro} = E\left[(1+\mu)A_o - (1-\mu)\frac{B_o}{r^2}\right]; \quad \sigma_{\theta o} = E\left[(1+\mu)A_o + (1-\mu)\frac{B_o}{r^2}\right]$$

Where

$$A_i = -\frac{b^2 t_o S}{E(1+\mu)}\left[\frac{1}{a^2(1+\mu)(t_i - t_o) - b^2 t_i(1+\mu) - b^2 t_o(1-\mu)}\right]$$

$$B_i = -\frac{a^2 b^2 t_o S}{E(1-\mu)}\left[\frac{1}{a^2(1+\mu)(t_i - t_o) - b^2 t_i(1+\mu) - b^2 t_o(1-\mu)}\right]$$

$$A_o = \frac{S}{2E(1+\mu)}$$

$$B_o = \frac{b^2 S}{2E(1-\mu)}\left[\frac{b^2(1-\mu)(t_i - t_o) - a^2 t_i(1-\mu) - a^2 t_o(1+\mu)}{a^2(1+\mu)(t_i - t_o) - b^2 t_i(1+\mu) - b^2 t_o(1-\mu)}\right]$$

Stresses due to radial component ($\frac{1}{2}S$ Cos 2θ) and shearing component ($-\frac{1}{2}S$ Sin 2θ):

$$\sigma_{ri} = -\left(2A_i + \frac{6}{r^4}C_i + \frac{4}{r^2}D_i\right)\cos 2\theta$$

$$\sigma_{ro} = -\left(2A_o + \frac{6}{r^4}C_o + \frac{4}{r^2}D_o\right)\cos 2\theta$$

$$\sigma_{\theta i} = \left(2A_i + 12B_i r^2 + \frac{6}{r^4}C_i\right)\cos 2\theta$$

$$\sigma_{\theta o} = \left(2A_o + 12B_o r^2 + \frac{6}{r^4}C_o\right)\cos 2\theta$$

The constants are given on the following pages.

$$\text{NUM. } A_i = -\frac{S}{4}\left\{\frac{36}{a^6 b^9}\left[t_o^2(3-\mu)+t_i t_o(1+\mu)\right]-\frac{108}{a^2 b^8}\left[t_o^2(1+\mu)-t_i t_o(1+\mu)\right]\right.$$

$$\left.+\frac{144}{b^{10}}\left[t_o^2(1+\mu)-t_i t_o(1+\mu)\right]\right\}$$

$$\text{Num. } B_i = -\frac{S}{4}\left\{-\frac{72}{a^9 b^8}\left[t_i t_o(1+\mu)-t_o^2(1+\mu)\right]+\frac{72}{a^2 b^{10}}\left[t_i t_o(1+\mu)-t_o^2(1+\mu)\right]\right\}$$

$$\text{Num. } C_i = -\frac{S}{4}\left\{\frac{36}{a^2 b^9}\left[t_i t_o(1+\mu)+t_o^2(3-\mu)\right]+\frac{36a^2}{b^8}\left[t_o^2(1+\mu)-t_i t_o(1+\mu)\right]\right\}$$

$$\text{Num. } D_i = -\frac{S}{4}\left\{-\frac{72}{a^9 b^9}\left[t_i t_o(1+\mu)+t_o^2(3-\mu)\right]+\frac{72a^2}{b^{10}}\left[t_i t_o(1+\mu)-t_o^2(1+\mu)\right]\right\}$$

$$\text{Num. } A_o = -\frac{S}{4}\left[\text{Denominator}\right]$$

$$\text{Num. } B_o = 0$$

$$\text{Num. } C_o = -\frac{S}{4}\left\{-\frac{9}{a^6}\left[2t_i t_o(1-\mu^2)+t_i^2(1+\mu)^2-t_o^2(3-\mu)(1+\mu)\right]\right.$$

$$-\frac{36}{a^9 b^2}\left[2t_i t_o(2+\mu+\mu^2)-t_i^2(1+\mu)^2-t_o^2(3+\mu^2)\right]$$

$$+\frac{18}{a^2 b^9}\left[2t_i t_o(7+6\mu+3\mu^2)-3t_i^2(1+\mu)^2-3t_o^2(1+\mu)^2\right]$$

$$-\frac{36}{b^6}\left[2t_i t_o(1+\mu)^2-t_i^2(1+\mu)^2-t_o^2(1+\mu)^2\right]$$

$$\left.-\frac{9a^2}{b^8}\left[2t_i t_o(1-\mu^2)+t_i^2(1+\mu)^2-t_o^2(3-\mu)(1+\mu)\right]\right\}$$

NUM $D_o = -\frac{S}{4}\Big\{\frac{18}{a^6b^2}\Big[2t_it_o(1-\mu^2)+t_i^2(1+\mu)^2-t_o^2(3-\mu)(1+\mu)\Big]$

$$+\frac{72}{a^4b^4}\Big[2t_it_o\mu(1+\mu)-t_i^2(1+\mu)^2-t_o^2(3+\mu^2)\Big]$$

$$-\frac{108}{a^2b^6}\Big[2t_it_o(1+\mu)^2-t_i^2(1+\mu)^2-t_o^2(1+\mu)^2\Big]$$

$$+\frac{72}{b^8}\Big[2t_it_o(1+\mu)^2-t_i^2(1+\mu)^2-t_o^2(1+\mu)^2\Big]$$

$$+\frac{18a^2}{b^{10}}\Big[2t_it_o(1-\mu)^2+t_i^2(1+\mu)^2-t_o^2(3-\mu)(1+\mu)\Big]\Big\}$$

DENOMINATOR $= \frac{9}{a^6b^4}\Big[2t_it_o(5-2\mu+\mu^2)+t_i^2(3-\mu)(1+\mu)+t_o^2(3-\mu)(1+\mu)\Big]$

$$+\frac{36}{a^4b^6}\Big[2t_it_o\mu(1-\mu)-t_i^2(3-\mu)(1+\mu)+t_o^2(3+\mu^2)\Big]$$

$$-\frac{54}{a^2b^8}\Big[2t_it_o(1-\mu^2)-t_i^2(3-\mu)(1+\mu)+t_o^2(1+\mu)^2\Big]$$

$$+\frac{36}{b^{10}}\Big[2t_it_o(1-\mu^2)-t_i^2(3-\mu)(1+\mu)+t_o^2(1+\mu)^2\Big]$$

$$+\frac{9a^2}{b^{12}}\Big[(t_i-t_o)^2(3-\mu)(1+\mu)\Big]$$

APPENDIX B

CALCULATIONS

Symbols:

A - area, $in.^2$

E - modulus of elasticity, psi.

σ - stress, psi.

ϵ - strain, in/in.

Ld - load, lbs.

G.F. - gage factor.

S - applied stress, psi.

θ - angle measured from the direction of loading

a - radius of cutout, in.

b - radius of doubler, in.

tc - thickness of core, in.

to - thickness of sheet, in.

ti - thickness of sheet and doubler, in.

k - ratio of radii, b/a.

q - ratio of thicknesses, to/ti.

$\sigma_{\theta a}$ - tangential stress at hole due to ($\frac{1}{2}$S) component of radial stress, psi.

$\sigma_{\theta b}$ - tangential stress at hole due to ($\frac{1}{2}$SCos 2θ) component of radial stress and ($-\frac{1}{2}$S Sin 2θ) component of shear stress, psi.

$\sigma_\theta = \sigma_{\theta a} + \sigma_{\theta b}$ - total tangential stress at hole in terms of applied stress, S.

APPENDIX B

CALCULATIONS

Symbols:

A - area, in.^{2}

E - modulus of elasticity, psi.

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Ld - load, lbs.

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tc - thickness of core, in.

to - thickness of sheet, in.

ti - thickness of sheet and doubler, in.

k - ratio of radii, b/a.

q - ratio of thicknesses, to/ti.

$\sigma_{\theta a}$ - tangential stress at hole due to $(\frac{1}{2}S)$ component of radial stress, psi.

$\sigma_{\theta b}$ - tangential stress at hole due to $(\frac{1}{2}S\,\text{Cos}\ 2\theta)$ component of radial stress and $(-\frac{1}{2}S\ \text{Sin}\ 2\theta)$ component of shear stress, psi.

$\sigma_\theta = \sigma_{\theta a} + \sigma_{\theta b}$ - total tangential stress at hole in terms of applied stress, S.

E' - $(3 - \mu)$

F - $(1 + \mu)$

G - $(1 - \mu^2)$

H - $(1 + \mu)^2$

I - $(3 + 2\mu - \mu^2)$

J - $(5 - 2\mu + \mu^2)$

L - $(\mu - \mu^2)$

M - $(3 + \mu^2)$

PERCENT OF LOAD TAKEN BY METALITE FACES:

$$Ld_{total} = Ld_{faces} + Ld_{core}$$

$$Ld_t = A_f \sigma_f + A_c \sigma_c$$

$$Ld_t = A_f E_f \epsilon_f + A_c E_c \epsilon_c$$

Metalite - 18 in. wide
to = .032 in.
tc = .50 in.
E_f = 10.3×10^6 psi.
E_c = $.0166 \times 10^6$ psi.

$$\frac{Ld_f}{Ld_t} = \frac{A_f E_f \epsilon_f}{A_f E_f \epsilon_f + A_c E_c \epsilon_c} \qquad \epsilon_f = \epsilon_c$$

$$\frac{Ld_f}{Ld_t} = \frac{(2)(18)(.032)(10.3)}{(2)(18)(.032)(10.3) + (1)(18)(.5)(.0166)}$$

$$\frac{Ld_f}{Ld_t} = .987$$

Faces take 98.7 percent of the impressed load.

THEORETICAL APPLIED STRESS:

zero loading = 100 lbs.
first loading = 6100 lbs.
ΔLd = 6000 lbs.

$A_f = 1.152$ in^2

$$\text{stress} = \frac{\Delta Ld}{A_f} = \frac{6000}{1.152} = 5220 \text{ psi.}$$

STRESS OBTAINED FROM STRAIN METER READINGS:

Load	Meter
100 lbs.	0
6100 lbs.	1070
Δread =	1070

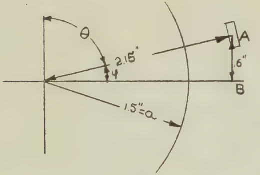
Metalite - No Doubler
3 in. cutout
Gage No. 7; SR-4, A-19
Gage Factor = 1.63
E = 10.3×10^6 psi.

$$\epsilon = \frac{(\Delta \text{read})(2.05)(10^{-6})}{(\text{G.F.})}$$

$$\epsilon = \frac{(1070)(2.05)(10^{-6})}{(1.63)} = 1346 \times 10^{-6} \text{ in./in.}$$

$$\sigma = \epsilon E = (1346)(10.3) = 13880 \text{ psi.}$$

CORRECTION FOR STRAIN GAGES NOT ON THE AXIS OF SYMMETRY:



Riveted Doubler
3 in. cutout

$$\varphi = \sin^{-1} \frac{.6}{2.15} = 16°15'$$

$$\theta = 90 - \varphi = 73°45'$$

$$\sigma_\theta = \frac{S}{2}\left(1 + \frac{a^2}{r^2}\right) - \frac{S}{2}\left(1 + \frac{3a^4}{r^4}\right) \text{Cos } 2\theta$$

$$\sigma_{\theta_A} = \frac{S}{2}\left(1 + \frac{(1.5)^2}{(2.15)^2}\right) - \frac{S}{2}\left(1 + \frac{3(1.5)^4}{(2.15)^4}\right) \text{Cos } 2(73°45')$$

$$\sigma_{\theta_A} = S(.7435) - S(.855)(-.844) = 1.4655S$$

$$\sigma_{\theta_B} = S(.7435) - S(.855)(-1.0) = 1.5985S$$

$$\sigma_{\theta_B} = \frac{(1.5985)}{(1.4655)} \sigma_{\theta_A} = 1.09\,\sigma_{\theta_A}$$

PERCENT OF LOAD TAKEN BY THE FRONT FACE:

Meter Readings

Gage	Front	Rear
1	990	
2		950
3	840	
4		850
5	940	
6		1000
sum	2770	2800

Metalite - no doubler
3 in. cutout
Gages - SR-4,A-11
Gage Factor = 2.08
Load = 10100 lbs.
$E = 10.3 \times 10^6$ psi.

$$\text{Avg. } \sigma \text{ front} = \frac{(2770)}{3}\frac{(2.05)}{(2.08)}(10.3) = 9100 \text{ psi.}$$

$$\text{Avg. } \sigma \text{ rear} = (2800)\frac{(2.05)}{(2.08)}(10.3) = 9200 \text{ psi.}$$

$$\text{Avg. } Ld_f = \sigma_f A_f = 9100A_f$$

$$\text{Avg. } Ld_r = \sigma_r A_r = 9200A_r \qquad A_f = A_r$$

$$Ld_t = Ld_f + Ld_r = 18300A$$

$$\frac{Ld_f}{Ld_t} = \frac{9100A}{18300A} = .4975$$

The front face takes 49.75 percent of the load.

THEORETICAL MAXIMUM STRESS AT CUTOUT:

μ = .3
a = 1.5 in.
b = 4.5 in.
to = .032 in.
ti = .064 in.

$$k = \frac{b}{a} = \frac{4.5}{1.5} = 3$$

$$k^2 = 9$$

$$k^4 = 81$$

$$k^6 = 729$$

$$k^8 = 6561$$

$$q = \frac{to}{ti} = \frac{.032}{.064} = \frac{1}{2}$$

$$\frac{1}{q} = 2 \; ; \quad \frac{1}{q^2} = 4$$

$E' = (3-\mu) = 2.7$; $F = (1+\mu) = 1.3$; $G = (1-\mu^2) = .91$

$H = (1+\mu)^2 = 1.69$; $I = (3+2\mu-\mu^2) = 3.51$

$J = (5-2\mu+\mu^2) = 4.49$; $L = (\mu-\mu^2) = .21$

$M = (3+\mu^2) = 3.09$

$$\sigma_{\theta_a} = \frac{-2S}{F\left[\frac{1}{K^2}\left(\frac{1}{q}-1\right)-\left(\frac{1}{q}+\frac{G}{H}\right)\right]}.$$

$$\sigma_{\theta_b} = \frac{-8\left[\left(\frac{3F}{K^4}-\frac{2F}{K^6}\right)\left(\frac{1}{q}-1\right)-\left(\frac{F}{q}+E\right)\right]S}{\frac{4}{K^2}\left(\frac{2L}{q}-\frac{I}{q^2}+M\right)-\left(\frac{6}{k^4}-\frac{4}{k^6}\right)\left(\frac{2G}{q}-\frac{I}{q^2}+H\right)+\frac{1}{k^8}\left(\frac{I}{q^2}-\frac{2I}{q}+I\right)\cdots}$$

$$\cdots\frac{\qquad}{\left(+\frac{2J}{q}+\frac{I}{q^2}+I\right)}.$$

$$\sigma_\theta = \sigma_{\theta_a} + \sigma_{\theta_b}$$

Substitution of the above constants in the formulas yield:

$\sigma_{\theta_a} = .634\ S$

$\sigma_{\theta_b} = 1.341\ S$

$$\sigma_\theta = .634S + 1.341S = 1.975S$$

Thesis 17331
L63

Lieber

Stress concentrations around reinforced circular cutouts in metalite panels in tension.

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